$$\text{Area } ACB = \text{area } ADB = \tfrac{1}{2}AB.\,CK = \frac{a^2\sqrt{2}}{2} = A.$$

$$\text{Area } AEB = \tfrac{1}{2}AB.\,EO = \frac{3a^2\sqrt{2}}{8} = A'.$$

$$V = \text{required volume} = 2(2\pi GH.A) - 2\pi EO.\,\frac{A'}{3}.$$

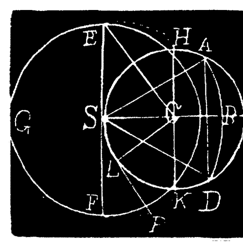
$$\therefore\ V = \frac{4a^3\pi}{3\sqrt{3}} - \frac{a^3\pi\sqrt{3}}{8} = \frac{23a^3\pi}{24\sqrt{3}} = \frac{23a^3\pi\sqrt{3}}{72}.$$

37. Proposed by J. A. CALDERHEAD, Superintendent of Schools, Limaville, Ohio.

A man ties two mules—one to the outside of a circular wall, the other to the inside. If the lengths of the ropes of each is one-fourth the circumference of the wall, and both together can graze over one acre of ground: find the circumference of the wall.

I. Solution by COOPER D. SCHMITT, Professor of Mathematics in the University of Tennessee, Knoxville, Tennessee.

Let S be the point where the mules are fastened. The mule grazing on the outside, grazes over the semi-circle $EGF+HSE$ and FSP. The mule on the inside grazes over the segment $SHA+$ segment $SKD+$ sector ASD.



We must find the area of these different portions and their sum equals one acre or 160 sq. rods.

1. To find the area of sector $ABDSA$. Since $AS=\frac{\pi r}{2}$, we have $\cos ASB=\frac{1}{4}\pi$. $\therefore$ $ASB=38^\circ\ 15'$, and area of sector $=\frac{76\frac{1}{2}}{360}.\pi.\frac{\pi^2r^2}{4}$.

2. To find area of segment SHA. $\angle SCA=103^\circ\ 30'$. Segment $SHA=$sector $SHAC-\triangle ASC$ $=\frac{103\frac{1}{2}}{360}\pi r^2-\frac{1}{2}r^2\sin 103\frac{1}{2}^\circ$. Segment $DSK=$segment SAH.

3. Area of $EGF=\frac{1}{2}\pi\left(\frac{\pi^2r^2}{4}\right)$.

4. Area of involute $ESHA=$area $CHE+$area $ECS-$area SCH $=$(by the Calculus) $\frac{SC^3}{6r}+\frac{1}{2}SC.r-\frac{1}{4}\pi r^2$. But $SC=\frac{1}{2}\pi r$. $\therefore$ involute $ESHA=(\frac{1}{2}\pi r)^3\div 6r+\frac{1}{2}.\frac{1}{2}\pi r.r-\frac{1}{4}\pi r^2=\frac{1}{48}\pi^3r^2$.

The area of involute $FSLK=$area of involute $ESHA$.

Combining these different areas, we have

$$r^2\left[\frac{153}{720}\,\frac{\pi^3}{4}+\frac{207}{360}\pi-\sin 103\tfrac{1}{2}^\circ+\frac{\pi^3}{8}+\frac{n^3}{24}\right]=160.$$

$$r^2\left[\frac{51\pi^3}{960}+\frac{207}{360}\pi-\ .9724+\frac{\pi^3}{6}\right]=160,$$

$$r^2\left[\frac{211}{960}\pi^3+\frac{207}{360}\pi-.9724\right]=160,$$

$r^2[6.8148+1.8064-.9726]=160$, $r^2(7.6486)=160$, $r^2=20.91$, $r=4.57$. $2\pi r=28.714+$ the circumference of the wall.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Vice President and Professor of Mathematics in the Texarkana College, Texarkana, Arkansas.

Let ASD, in Fig. above, be the given wall. S the point where the mules are fastened, a=radius of wall, $\varphi=\angle KCL$, $PL=\rho$,=radius of curvature of involute KPF, $\theta=\angle ASC$.

We now have the three areas to find:— (1) area $SHABDKS$, (2) the the two equal involute areas SHE and SKF, (3) area semi-circle EGF. $SA=SE=\frac{1}{2}\pi a$.

$\therefore$ Area semi-circle $EGE=\frac{1}{8}\pi^3a^2\ldots.(1)$.

Area of an element between two consecutive radii of curvature is $dA=\frac{1}{2}\rho^2d\varphi=\frac{1}{2}a^2\varphi^2d\varphi$, since $\rho=a\varphi$.

$$\therefore \text{ Area } (SHE+SKF)=a^2\int_0^{\frac{\pi}{2}}\varphi^2d\varphi=\tfrac{1}{24}\pi^3a^2\ldots.(2).$$

Area common to both circles$=a^2(\pi+2\theta\cos 2\theta-\sin 2\theta)$, but $2a\cos\theta=\frac{1}{2}a\pi$, $\therefore\cos\theta=\frac{1}{4}\pi$.

$\therefore$ Area common to both circles

$$=a^2\left\{\pi+\tfrac{1}{4}(\pi^2-8)\cos^{-1}\frac{\pi}{4}-\frac{\pi}{8}\sqrt{16-\pi^2}\right\}\ldots.(3).$$

$$\therefore a^2\left\{\tfrac{1}{6}\pi^3+\pi+\tfrac{1}{4}(\pi^2-8)\cos^{-1}\frac{\pi}{4}-\frac{\pi}{8}\sqrt{15-\pi^2}\right\}=160 \text{ sq. rods.}$$

$$\therefore a^2\left(7.337+.4674\cos^{-1}\frac{\pi}{4}\right)=160 \text{ sq. rods.}$$

$\therefore 7.64896a^2=160$, $a=4.5736$ rds.

$2\pi a=28.7368$ rods,=circumference required.

Also solved by *A. H. BELL* and *F. P. MATZ.*

PROBLEMS.

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45. Proposed by Dr. GEORGE LILLEY, Portland, Oregon.

A fly starts from a point in the circumference of a table, 3 feet in diameter, and travels uniformily along the diameter to a point in the circumference of the table directly opposite the starting point. The table moves uniformily to the right about a center axis in such manner that it makes one complete revolution while the fly passes over its diameter. Find the absolute path described by the fly and the ratio of rates of movement of the table and the fly.